

Introduction:

The human gastrointestinal (GI) system consists of a large intestine and is made up of the colon and rectum. The colon cancer starts in the colon and the rectum cancer starts in the rectum, and collectively they are called **colorectal cancer (CRC)**.

- The large intestine is the preferred location of carcinogenesis, providing a lot of potential for cancer because of its propensity to self-renew as well as the large number of cell divisions.

Literature Review:

- Tomlinson et al. [1] proposed a mathematical model that takes into account of PCD failure or differentiation failure. They assigned crypt cells to one of these three different compartments: SCs, SDCs, and DCs.
- Tomlinson and Bodmer [1] were improved by Johnston et al. where they assumed the cell population period increased over the cell cycle time and took into consideration the asynchronicity as well.

Motivation:

The CRC is the 3rd most commonly diagnosed cancer type in humans in addition to being the second biggest cause of cancer-related deaths. Its death rate increased 1 percent per year since mid-2000s.

Fractional Integral Transform:

Let $f(t)$ be an integrable function defined for all $t \geq 0$, $\gamma(t)$ is a normalization function, $\phi^\alpha(s) \neq 0$ and $\psi(s)$ are positive real functions, then the new integral transform is defined as:

$$S\{f(t); s\} = F(s) = \phi^\alpha(s) \int_0^\infty \gamma(t) e^{-\psi(s)t} f(t) dt.$$

Provided that the integral exists for some $\psi(s)$ and inverse of the transform is the original function.

Mathematical Models:

This mathematical model describing the colonic crypt cell population behavior and its role in the CRC:

$$\begin{aligned} Y_1'(t) &= (\lambda_3 - \lambda_1 - \lambda_2)Y_1(t), \\ Y_2'(t) &= (\lambda_6 - \lambda_4 - \lambda_5)Y_2(t) + \lambda_2 Y_1(t), \\ Y_3'(t) &= \lambda_5 Y_2(t) - \mu Y_3(t). \end{aligned}$$

With initial conditions ICs,

$$Y_1(0) = C_1, Y_2(0) = C_2, Y_3(0) = C_3.$$

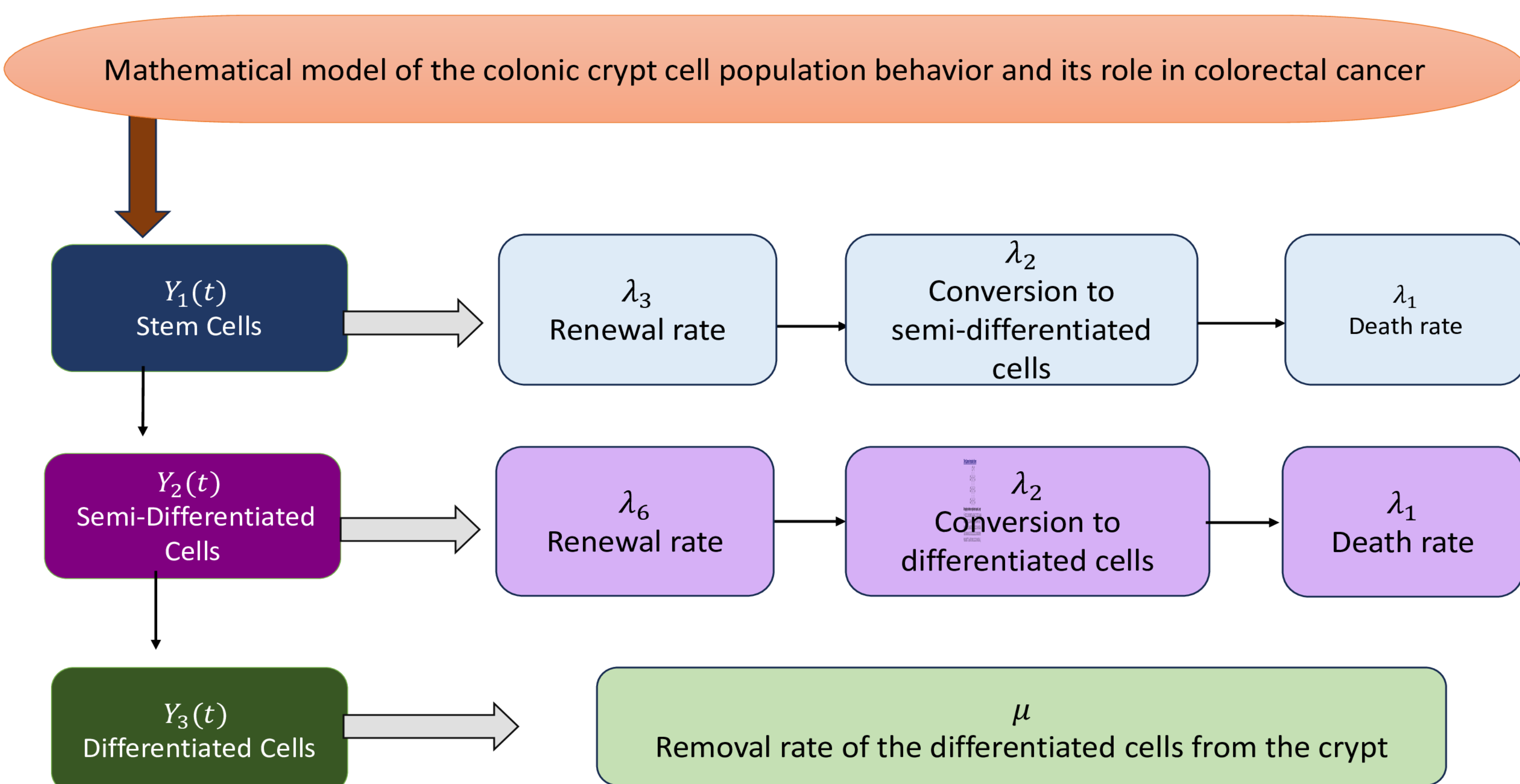


Figure 1, Shows the natural explanation of the parameters of the model.

Solution of Mathematical Model:

Using our proposed transform, we devised a new technique **TLUDM**. Using this method, we have results as follows:

$$\begin{aligned} Y_1(t) &= C_1 e^{wt}, \\ Y_2(t) &= \left\{ C_2 + \frac{C_1 \lambda_2}{(l-w)} \right\} e^{lt} + \frac{C_1 \lambda_2}{(l-w)} e^{wt}, \\ Y_3(t) &= \left(C_3 - \frac{C_2 \lambda_5}{(l+\mu)} + \frac{C_1 \lambda_2 \lambda_5}{\mu(l+w+\mu)+lw} \right) e^{-\mu t} \\ &\quad + \left(\frac{C_2 \lambda_5}{(l+\mu)} + \frac{C_1 \lambda_2 \lambda_5}{(\mu(l+w+\mu)+lw)} \left(\frac{w+\mu}{l-w} \right) \right) e^{lt} \\ &\quad + \left(\frac{C_1 \lambda_2 \lambda_5}{(\mu(l+w+\mu)+lw)} \left(\frac{\mu+l}{w-l} \right) \right) e^{wt}. \end{aligned}$$

Graphical View:

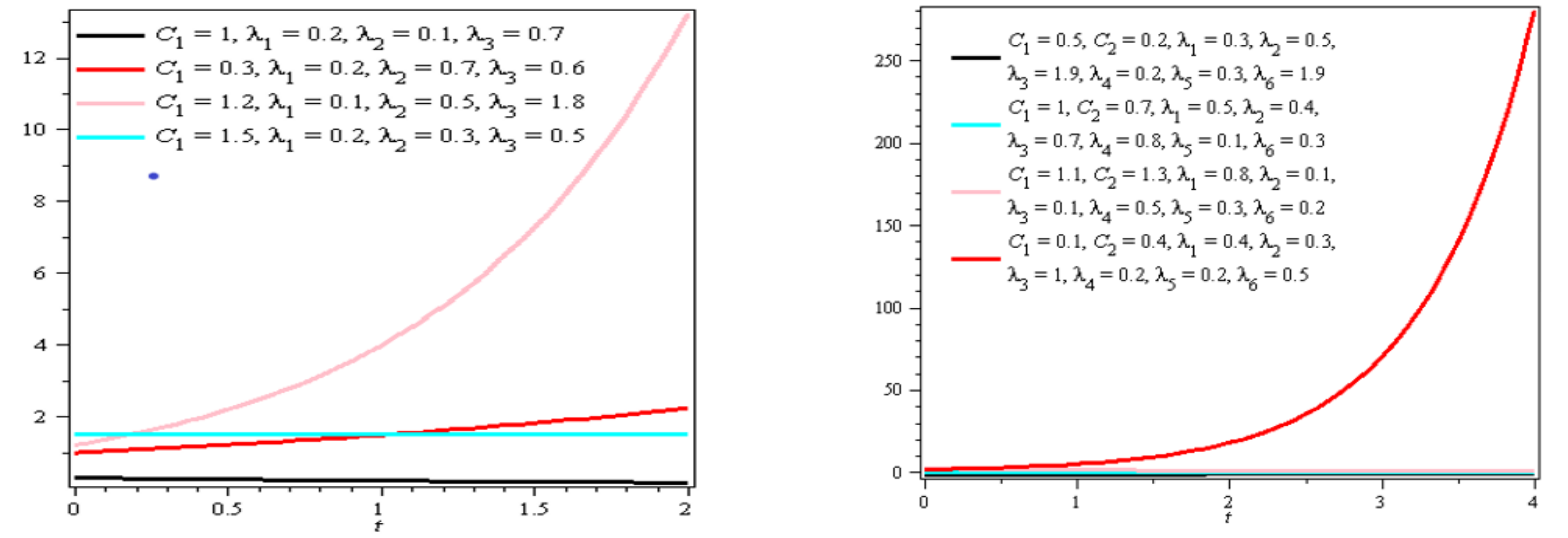


Figure 2, Graphs of SCs and SDCs are presented, respectively.

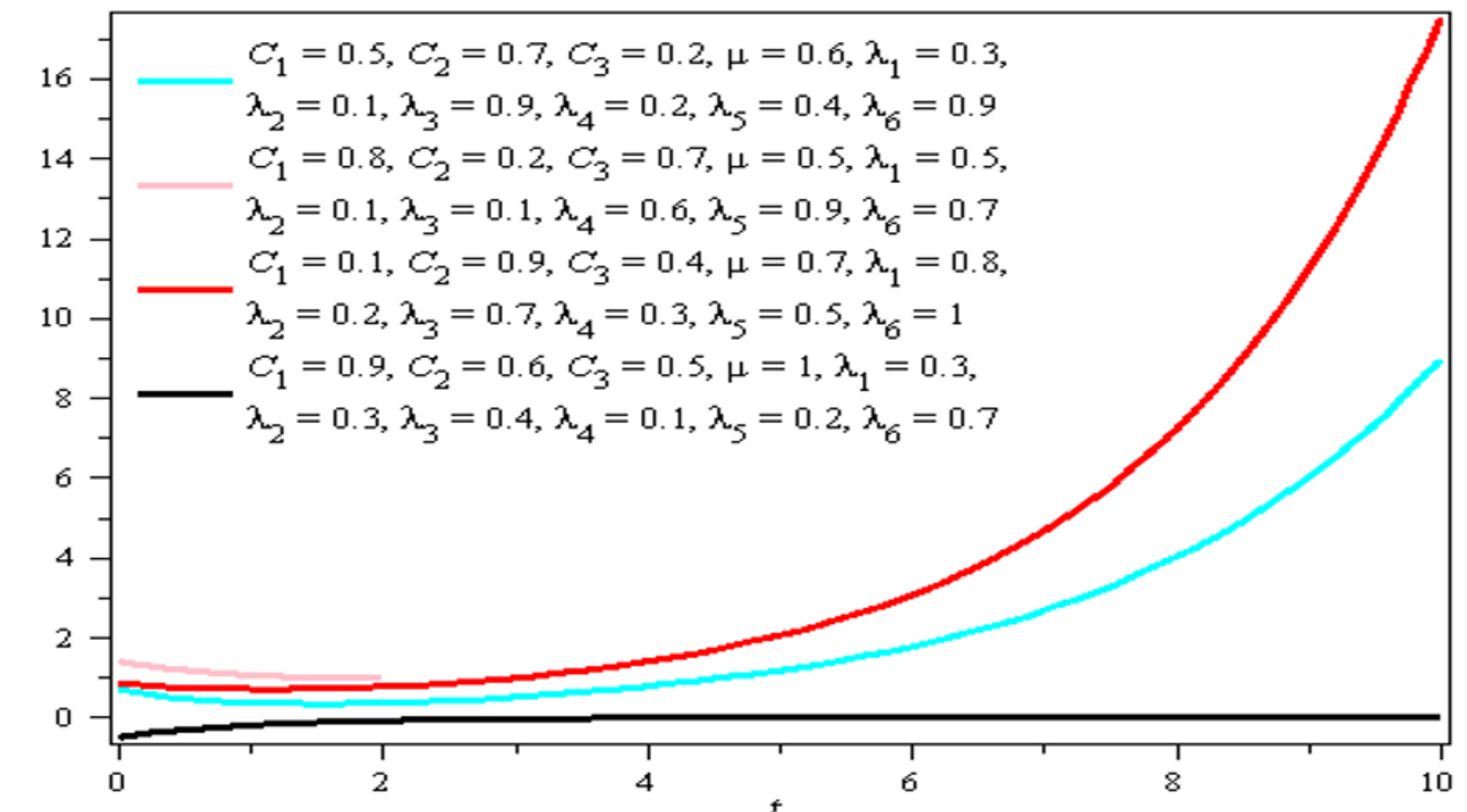


Figure 3, Graph of DCs is presented.

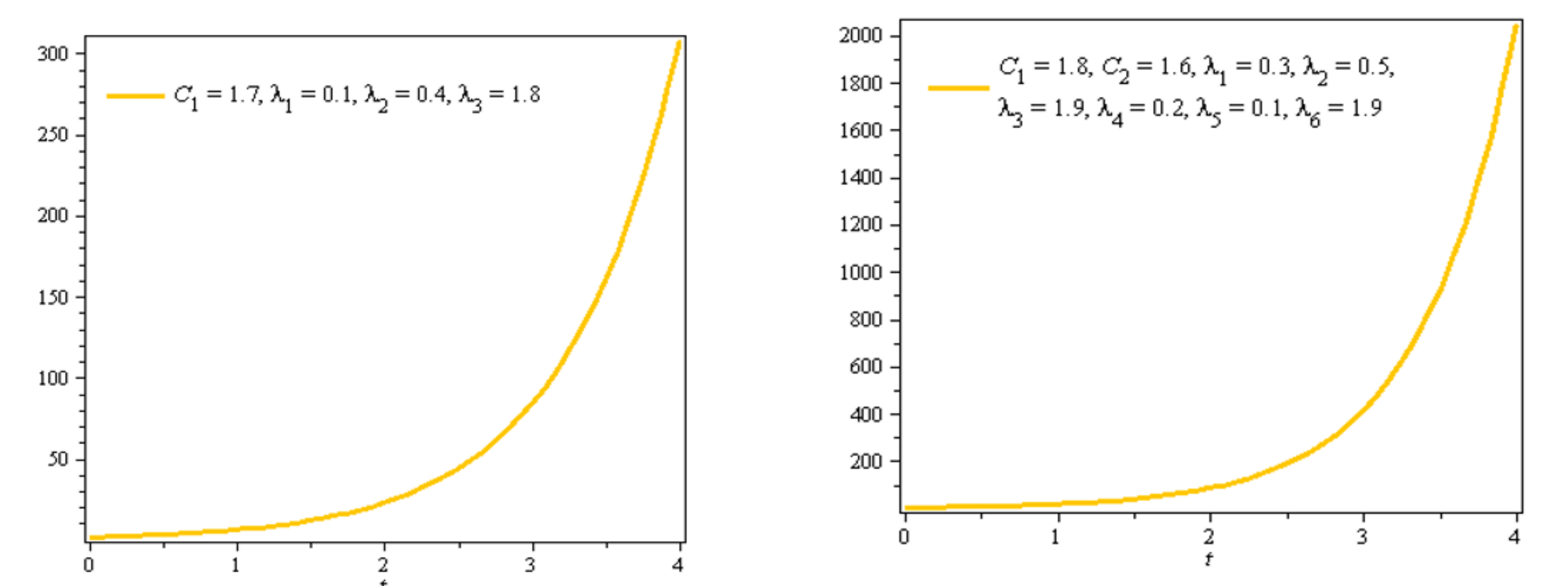


Figure 4, Graphs of day-wise (unbounded) growth of SCs and SDCs.

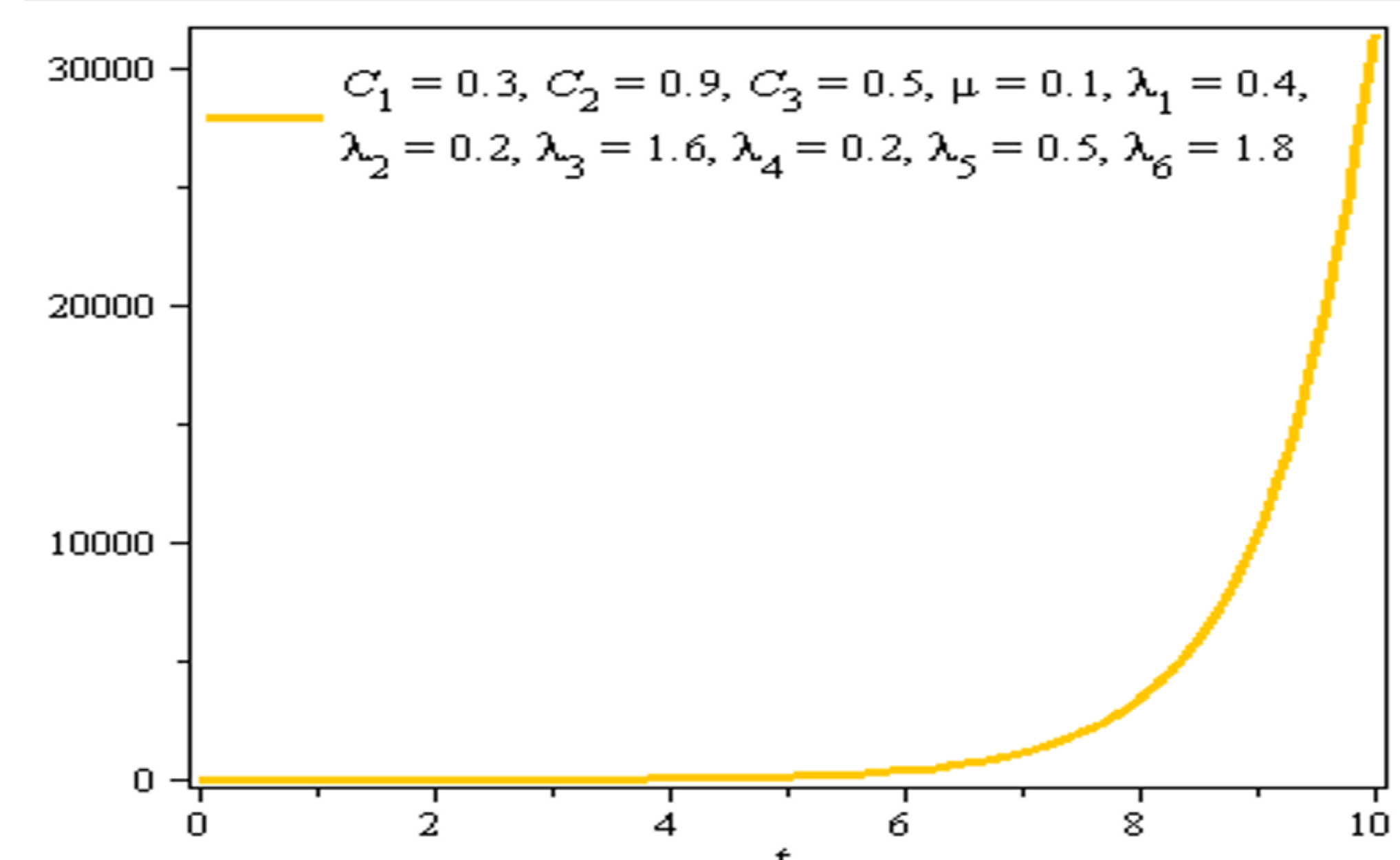


Figure 5, Graph of day-wise (unbounded) growth of DCs.

Conclusion:

- When the rate of SCs and SDCs is greater than one, the graph goes towards exponential growth gradually, and for all the other values, the solution will be negative, and a steady state can be viewed.

- When the rate of both SCs and SDCs is negative, graph slowly increases before becoming stable. When SC's rate is negative; the solution will halt at two. When just SDCs are negative, the graph steadily increases.

- The actual number of SCs reproduces in 4 days, and from day 5 unbounded growth will come into view. Similarly, the actual SDCs in the colon crypt will be renewed, but from day 5 an enlarged unbounded growth will happen.

- For almost the first 4 days, the DCs remain stable, and from the 5th day, they start to grow. On the 10th day it goes to exponential growth, and more than 30,000 cell formation come into view, which leads to mutation (**basis of tumorigenesis**).

Acknowledgments:

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References:

- Johnston, M. D., Edwards, C. M., Bodmer, W. F., Maini, P. K. & Chapman, S. J. (2007). Mathematical modeling of cell population dynamics in the colonic crypt and in colorectal cancer. *Proceedings of the National Academy of Sciences*, 104(10):4008-4013.
- Younis, S., Saba, N., Mustafa, G., Asghar, M., & Khan, F. (2024). A generalized fractional integral transform for solutions of fractional burger's equation. *Punjab University Journal of Mathematics*, 56(10).